# ASSIGNMENT SET – I

## Mathematics: Semester-I

## M.Sc (CBCS)

## Department of Mathematics

## Mugberia Gangadhar Mahavidyalaya

## PAPER - MTM-106

## Paper: Graph Theory

### Answer all the questions

1. Define graph.
2. Define self loop and parallel edge in a graph.
3. Define simple graph.
4. Define incidence and degree.
5. Define isolated vertex and pendant vertex and null graph.
6. State and proof hand shaking lemma.
7. Define isomorphic graph with example.
8. Define sub graph and its compliment.
9. Define spanning sub graph, edge disjoin sub graph, vertex disjoint sub graph.
10. Define self complementary graph.
11. Prove that in a self complementary the no of vertices has 4k or 4k+1.
12. Show that the maximum number of edges in a simple graph with in *n* vertices is $\frac{n(n-1)}{2}$.
13. Can a graph with eleven vertices be isomorphic to its complement?
14. Prove that a simple graph G with at least two vertices contain two vertices of same degree.
15. What is the maximum number of vertices in a graph with 41 edges and all the vertices at least degree three?
16. Define walk, path and circuits.
17. What is the length of a path?
18. Define connected and disconnected graph.
19. Prove that in a simple graph with *n* vertices and *K* components can have most $\frac{(n-K)(n-K+1)}{2}$ edges.
20. If a simple graph G with n vertices has more than $\frac{(n-2)(n-1)}{2}$ edges, then prove that G is connected.

21. Define intersection, union and ring sum of two graph.
22. Define decomposition of a graph.
23. Define complete and regular graph.
24. Define cycle, path and wheel graph.
25. Define bipartite graph and complete bipartite graph.
26. Show that maximum number of edges in complete bipartite graph of n vertices is $\frac{n^2}{4}$
27. What is the sum of the degree of the vertices of three regular graphs with n vertices?
28. Which of the platonic graphs have Hamiltonian circuits?
29. Which complete bipartite graphs are Hamiltonian as well as Eulerian?
30. For which values of n is the wheel $\boldsymbol{W_n}$ Hamiltonian?
31. Prove that if **G** is a bipartite graph with an odd number of vertices then **G** is not Hamiltonian?
32. Draw a graph that has both an Euler circuit and a Hamiltonian circuit
33. Draw a graph that has an Euler circuit but has no Hamiltonian circuit.
34. Draw a graph that has Hamiltonian circuit but has no Euler circuit.
35. Draw a graph that has Hamiltonian circuit but has no Euler circuit.
36. Draw a graph that has Hamiltonian circuit but has no Euler circuit.
37. Draw a graph that has neither an Euler circuit nor a Hamiltonian circuit.
38. Let G be bipartite graph with disjoint vertex sets $\boldsymbol{V_1}$ an$\boldsymbol{V_2}$d.show that if **G** has a Hamiltonian circuit then $\boldsymbol{V_1}$ and $\boldsymbol{V_2}$ have the same number of elements.
39. Prove that a graph **G** with n vertices has a Hamiltonian path if the sum of the degree of every pair of vertices $\boldsymbol{v_i}$ and $\boldsymbol{v_j}$ in **G** satisfies the condition

$$\mathbf{d}(\boldsymbol{v_i}) + \mathbf{d}(\boldsymbol{v_j}) \geq \boldsymbol{n} - \mathbf{1}$$

40. Show that the Petersen graph is non Hamiltonian.

End